IB Math SL 1-2 Study Guide for Semester 1 Final name KEY

Do your best. Show all of your work.
I recommend you work out the entire study guide on your own before I go over it in class.
I recommend you pay attention and correct any mistakes when I go over the study guide.
I recommend you rework the entire study guide again before the Final.
If you need extra practice on any topic, find problems on your old tests and homework.

1) Find the exact solutions of $\left(x + \frac{1}{x}\right) = 4$ · x

$x^2 + 1 = 4x$
$x^2 - 4x + 1 = 0$
$x = \frac{4 \pm \sqrt{16-4}}{2} = \frac{4 \pm \sqrt{12}}{2}$
$x = \frac{4 \pm 2\sqrt{3}}{2}$
$x = 2 \pm \sqrt{3}$
✓✓ w/graph y_1 LS y_2 RS

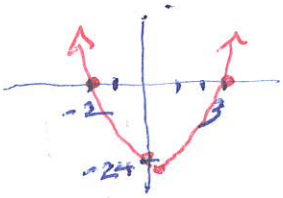
2) a) Write the expression $3x^2 + 18x + 20$ in the form $a(x - h)^2 + k$.

$3(x^2+6x+9) + 20 - 27$
$3(x+3)^2 - 7$

b) Hence find the exact solution of the equation $3x^2 + 18x + 20 = 0$
('hence' means you should use the previous part of the problem to do this part!)

$3(x+3)^2 - 7 = 0$
$3(x+3)^2 = 7$
$(x+3)^2 = 7/3$
$x+3 = \pm\sqrt{\frac{7}{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$
$x = \pm\frac{\sqrt{21}}{3} - 3$
✓✓ w graphs decimal solutions

3) Find the equation of the quadratic with x-intercepts of x=-2 and x=3 and y-intercept of y=-24.



$y = a(x+2)(x-3)$
$-24 = a(0+2)(0-3)$
$-24 = a(-6)$
$a = 4$
$y = 4(x+2)(x-3)$
or $y = 4x^2 - 4x - 24$
✓✓✓ graph - all 3 pts

4) The curve $y = x^2 + kx + 2$ touches the x-axis. Find the possible values of *k*.
(remember all the ways information can be given to you to find k: page 18)


so $\Delta = 0$
$b^2 - 4ac = 0$
$k^2 - 4(1)(2) = 0$
$k^2 - 8 = 0$
$k^2 = 8$
$k = \pm\sqrt{8}$
$k = \pm 2\sqrt{2}$
✓✓ w/both eqs w/k in graph

5) Solve the equation $5^{3x+1} = 15$, giving your answer in the form $\frac{\log(a)}{\log(b)}$ & *a* and *b* are integers.

~~$\log_5 15 = 3x+1$~~
~~$\log_5(15) - 1 = x$~~
~~$\frac{3}{3}$~~
~~$x = \frac{1}{3}(\log_5(15) - \log$~~
not working for form so try something else

$5^{3x+1} = 15$
$\log 5^{3x+1} = \log 15$
$(3x+1)\log 5 = \log 15$
$3x\log 5 + \log 5 = \log 15$
$3x\log 5 = \log 15 - \log 5$
$x = \frac{\log 15 - \log 5}{3\log 5}$
$x = \frac{\log\left(\frac{15}{5}\right)}{\log 5^3} = \frac{\log 3}{\log 125}$

6) Given a=log(x), b=log(y), and c=log(z), find an expression in terms of a, b, and c for

$\log\left(\frac{10xy^2}{\sqrt{z}}\right)$

$\log 10 + \log x + 2\log y - \frac{1}{2}\log z$
$1 + a + 2b - \frac{1}{2}c$

✓redo ✓OR a=2 b=3 c=4 ✓ok ✓decimal in orig EQ

7) Given that $\log a + 1 = \log b^2$, express a in terms of b. (note: is it log(a+1) or log(a)+1 ?)

$\log a + \log 10 = \log b^2$

$\log(10a) = \log b^2$

$10a = b^2$

$a = \frac{b^2}{10}$

I8

✓ TEDO

TRY $b=3$ so $a = \frac{9}{10}$

PLUG in! ✓

8) Solve the equation $2\log_2 x - \log_2(x+1) = 3$, giving answer in simplified form.

$\log_2 x^2 - \log_2(x+1) = 3$

$\log_2\left(\frac{x^2}{x+1}\right) = 3$

$2^3 = \frac{x^2}{x+1}$

$8x+8 = x^2$

$0 = x^2 - 8x - 8$

$x = \frac{8 \pm \sqrt{64+32}}{2} = \frac{8 \pm \sqrt{96}}{2}$

$x = \frac{8 \pm 4\sqrt{6}}{2} = 4 \pm 2\sqrt{6}$

$x = 4 + 2\sqrt{6}$ $\quad x = 4 - 2\sqrt{6}$ EXTRANEOUS

✓ Graph LS=RS

9) Using the substitution $u=x^2$, solve the equation $x^4 - 5x^2 + 4 = 0$

$u^2 - 5u + 4 = 0$

$(u-4)(u-1) = 0$

$u = 4 \quad u = 1$

$x^2 = 4$ $\quad x = 2 \quad x = -2$

$x^2 = 1$ $\quad x = 1 \quad x = -1$

✓ graph - zeros

10) Using the appropriate substitution, **solve** the equation $2^{2x} - 5\cdot 2^x + 4 = 0$

Let $u = 2^x$

$(2^x)^2 = 2^{2x}$

$u^2 - 5u + 4 = 0$

$(u-4)(u-1) = 0$

$u = 4 \quad u = 1$

$2^x = 4$ $\quad x = 2$ ✓

$2^x = 1$ $\quad x = 0$ ✓

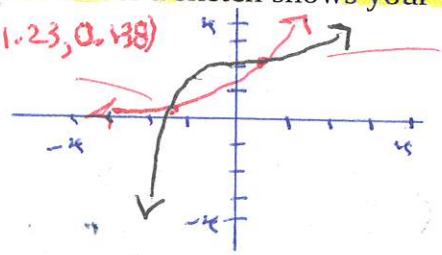
✓ in orig eq in calc or mental

11) Solve the equation $5^x = x^3 + 2$ (remember a sketch shows your 'work') 3 SIG FIGS

Tried -10 to 10 det
Then -4 to 4 det

$x \approx -1.23$

$x \approx 0.460$



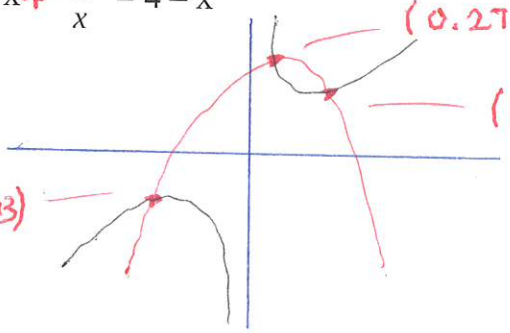
✓✓ in EQ

12) Find all the solutions of the equation $x + \frac{1}{x} = 4 - x^2$

$x \approx -2.65$

$x \approx 0.274$

$x \approx 1.38$



✓✓✓ in EQ.

13) A function is defined in the following table:

x	1	2	3	4	5	6	7	8	9
f(x)	7	1	6	4	2	4	9	8	3

a) Find $f \circ f(3)$. $f(6)=4$ — $F(3)$

b) Find $f^{-1}(9)$. $= 7$ — INPUT OF $f(x)$ — OUTPUT OF $f(x)$

14) If $f(x) = \frac{3x-1}{2x+3}$

a) Find an expression for $f^{-1}(x)$

b) State the domain of f(x) and the range of $f^{-1}(x)$.

$y = \frac{3x-1}{2x+3}$

$y(2x+3) = 3x-1$

$2xy + 3y = 3x - 1$

$3y + 1 = 3x - 2xy$

$3y + 1 = x(3-2y)$

$x = \frac{3y+1}{3-2y}$

$f^{-1}(x) = \frac{3x+1}{3-2x}$

$F(x)$ d: $x \neq -\frac{3}{2}$ ✓

$f^{-1}(x)$ r: $y \neq -\frac{3}{2}$

✓ w/ y_1 y_2 FLIPPED POINTS

15) Describe two transformations which transforms the graph of $y=x^2$ to the graph of:

a) $y = 3(x-2)^2$ (0,0) (1,1) (2,4) (2,0) (3,3) (4,12) ✓ ✓ ✓

Right 2, vert st factor 3, ~~Right 2~~

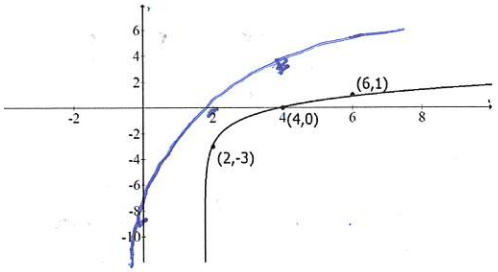
b) $y = 3x^2 - 2$ (0,-2) (1,1) (2,10) ✓ ✓ ✓

Vert st f. of 3 Then d. 2

✓ in calc

16) The diagram shows the graph of the function $y = f(x)$.

Sketch the graph of $y = 3f(x+2)$ V S 3 ← 2 2



$(2,-3) \rightarrow (0,-9)$

$(4,0) \rightarrow (2,0)$

$(6,1) \rightarrow (4,3)$

17) The fifth term of an arithmetic sequence is 25 and the eighth term is 64.

Find the first term and the common difference of the sequence. *(you must use a system of eq.s)*

$u_5 = 25$

$u_8 = 64$

$u_n = u_1 + d(n-1)$

$64 = u_1 + 7d$

$-(25 = u_1 + 4d)$

$39 = 3d$

$d = 13$

$25 = u_1 + 4(13)$

$u_1 = -27$

✓✓ in calc $y = -27 + 13(x-1)$ TABLE

18) The fifth term of an arithmetic sequence is 64 and the eighth term is 46.

a) Find the 30th term. *(you must use a system of eq.s)*

b) Find the sum of the first thirty terms.

CALC - TABLE

$u_5 = 64$

$u_8 = 46$

$46 = u_1 + 7d$

$-(64 = u_1 + 4d)$

$-18 = 3d$

$-6 = d$

$46 = u_1 + 7(-6)$

$u_1 = 88$

$u_n = 88 - 6(n-1)$ ✓

$u_{30} = 88 - 6(29)$

$u_{30} = -86$ ✓ TABLE

$S_{30} = \frac{30}{2}(88 + -86)$

$S_{30} = 30$

✓ added u_1

$\binom{n}{r}$ 1 8 28 56 70 56 28 8 1

19) The fifth term of a geometric sequence is 12 and the seventh term is 3. Find the two possible values of the sum to infinity of the series. *(you must use a system of eq.s to find 'r')*

$u_5 = 12$ $u_7 = 3$ $r = ?$ $u_1 = ?$

$u_n = u_1 \cdot r^{n-1}$

$\frac{3}{12} = \frac{u_1 \cdot r^6}{u_1 \cdot r^4}$

$\frac{1}{4} = r^2$

$r = \pm\frac{1}{2}$

$3 = u_1 \cdot \frac{1}{2}^6$

$u_1 = 192$

$S_\infty = \frac{192}{1-\frac{1}{2}}$ $S_\infty = \frac{192}{1--\frac{1}{2}}$

$S_\infty = 384$ or 128

20) The fifth term of a geometric sequence is 128 and the sixth term is 512.

a) Find the common ratio and the first term.

b) Which term has a value of 32768?

TABLE ✓✓✓

$u_5 = 128$ $u_6 = 512$ $r = 4$

$512 = u_1 \cdot 4^5$ $u_1 = \frac{1}{2}$

$32768 = \frac{1}{2}(4)^{n-1}$

$65536 = 4^{n-1}$

$\log_4 65536 = n-1$

$\log_4(65536) + 1 = n$

$n = 9$ ✓

TABLE or EQ

21) Find the term in which r=2 for $(x^3 + \frac{3}{x})^8$ $a = x^3$ $b = 3x^{-1}$ $n = 8$

$\binom{8}{2}(x^3)^6(3x^{-1})^2 = 28x^{18} \cdot 9x^{-2} = 252x^{16}$

22) Expand using the Binomial Theorem: $(x^3 + \frac{3}{x})^8$

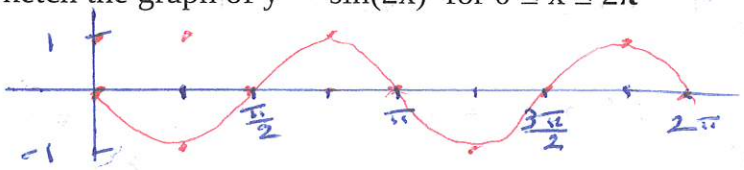
✓ x^4 pattern

$= 1(x^3)^8 + 8(x^3)^7(3x^{-1}) + 28(x^3)^6(3x^{-1})^2 + 56(x^3)^5(3x^{-1})^3 + 70(x^3)^4(3x^{-1})^4 + 56(x^3)^3(3x^{-1})^5 + 28(x^3)^2(3x^{-1})^6 + 8(x^3)(3x^{-1})^7 + (3x^{-1})^8$

$= x^{24} + 24x^{20} + 252x^{16} + 1512x^{12} + 5670x^8 + 13608x^4 + 20412 + 17496x^{-4} + 6561x^{-8}$

✓ calc

23) Sketch the graph of $y = -\sin(2x)$ for $0 \le x \le 2\pi$



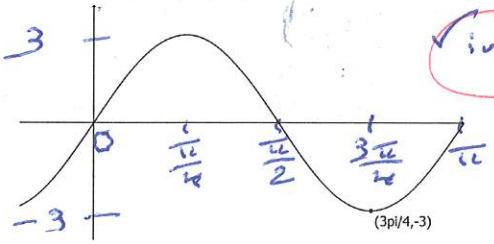
FLIPPED

Pd: $\frac{2\pi}{2} = \pi$

24) The graph shown has equation $y = a \cdot \sin(bx)$. Find the values of a and b.

$a = 3$ AMPLITUDE

$b = 2$ $\frac{2\pi}{2} = \pi$ = Pd.



✓ in calc

25) The depth of water in a harbor is modeled by the equation $d = 14 - 1.2\cos(\frac{pi\,t}{12})$, where d is measured in meters and t is the time in hours after midnight.

a) What is the first time after midnight at which the water depth is 14 meters?

b) What is the smallest possible depth?

c) Find the times, in the first 24 hours, when the depth of the water is less than 13.5 m.

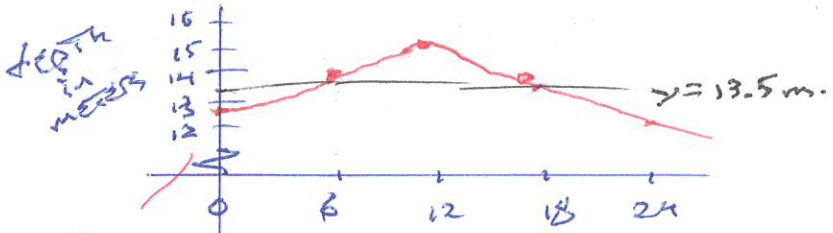
✓ EQ/graph

Pd: $\frac{2\pi}{\frac{\pi}{12}} = 24$ hours

(a) 6 a.m. (midline)

(b) 12.8 m. (14-1.2)

(c) $x = 4.36$ 4:21 a.m. $x = 19.64$ 7:38 a.m.

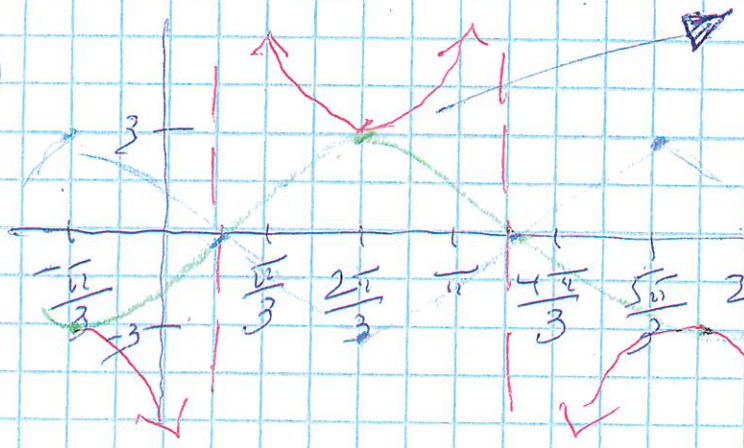


14 ± 1.2: (0, 12.8) (12, 15.2) (6, 14) (18, 14) (24, 12.8) # hours after mid.

SL Final SG 1st sem

#26-28

(26)



sec, now flip

-sec, now steam

$y = -3\sec\left(x+\frac{\pi}{3}\right)$ $\quad y = -3\frac{1}{\cos\left(x+\frac{\pi}{3}\right)}$

Pdi: 2π Flipped Amp: 3 Left $\frac{\pi}{3}$

Start: $-\frac{\pi}{3}$

End: $-\frac{\pi}{3}+2\pi = \frac{5\pi}{3}$

Halfway: $\frac{2\pi}{3}$ ✓

Sure if $x=\frac{2\pi}{3}$ for $\cos x = -\frac{1}{2}$

$=3$ ☺

(27) (a) $\cos\left(\frac{\pi}{6}\right)+\sec\left(\frac{\pi}{6}\right)$

$\frac{\sqrt{3}}{2}+\frac{2}{\sqrt{3}}$

$\frac{\sqrt{3}^2}{2\sqrt{3}}+\frac{2\cdot 2}{2\sqrt{3}}$

$\frac{3+4}{2\sqrt{3}}$

$\frac{7\sqrt{3}}{6}$ ✓

(b) $\sin^2\left(\frac{\pi}{4}\right)-1$

$\left(\frac{\sqrt{2}}{2}\right)^2-1$

$\frac{2}{4}-1$

$-\frac{1}{2}$

✓

(c) $\left(1+\cot\left(\frac{4\pi}{3}\right)\right)^2$

$\left(1+\frac{-1/2}{-\sqrt{3}/2}\right)^2$

$\left(1+\frac{1}{\sqrt{3}}\right)^2$

$\left(\frac{\sqrt{3}+1}{\sqrt{3}}\right)^2$

$\frac{3+2\sqrt{3}+1}{3}$

$\frac{4+2\sqrt{3}}{3}$ ✓

(28) (a) $\cos(x)-\frac{\csc(x)}{\cot(x)} = -\sin(x)\tan(x)$

$\cos(x)-\frac{1/\sin(x)}{\cos(x)/\sin(x)}$

$\cos(x)-\frac{1}{\cos(x)}$

$\frac{\cos^2 x}{\cos x}-\frac{1}{\cos(x)}$

$\frac{\cos^2 x-1}{\cos x}$

$-\frac{\sin^2 x}{\cos x}$

$-\sin x-\frac{\sin x}{\cos x}$

$-\sin x\tan x = \square$

LHS = RHS

(b) $(5n+1)^2-(5n-1)^2$

$25n^2+10n+1-(25n^2-10n+1)$

$20n$

$5(4n)$ $\square$

$n\in\mathbb{Z}^+$ so $4n\in\mathbb{Z}^+$

$\therefore 5(4n)$ is a multiple of 5 $n\in\mathbb{Z}^+$