

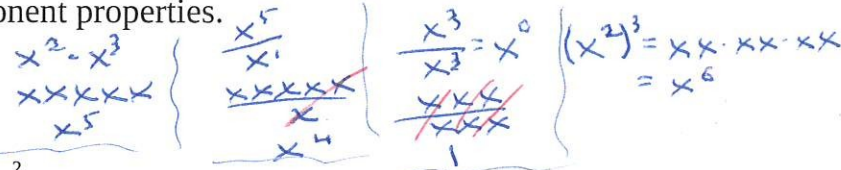
Algebra 3-4 honors STUDY GUIDE for FINAL 1<sup>st</sup> sem.

name KEY pd \_\_\_\_\_

I recommend you work out the entire study guide on your own before I go over it in class.  
 I recommend you pay attention and correct any mistakes when I go over the study guide.  
 I recommend you rework the entire study guide again before the Final.  
 If you need extra practice on any topic, find problems on your old tests and homework.

- 1) Give examples to demonstrate the exponent properties.

For example: Why does  $x^a \cdot x^b = x^{a+b}$ ?



- 2) Simplify.

a)  $\frac{4x^{-3}}{12xy^2} \cdot \frac{6y^{-5}z^2}{3x^{-2}y^2z^5}$

$= \frac{2x^{-3}y^{-5}z^2}{3x^1y^4z^5} = \frac{2}{3x^2y^9z^3}$

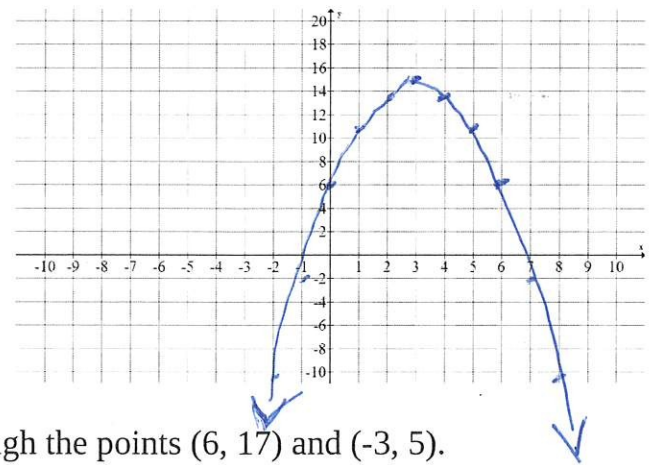
b)  $(5hj^0k^{10})^2 \cdot (4h^{-3}j^2k)^3$

$= 25h^2k^{20} \cdot 64h^{-9}j^6k^3$   
 $= \frac{1600j^6k^{23}}{h^7}$

- 3) Graph the quadratic on the given grid and find the characteristics.

Find the characteristics algebraically **and** check them with your calculator.

$y = -(x-3)^2 + 15$   
 $0 = -(x-3)^2 + 15$   
 $15 = (x-3)^2$   
 $x = \pm\sqrt{15} + 3$   
 x-intercepts:  $\approx (6.87, 0)$   $(-0.87, 0)$   
 y-intercept:  $(0, 6)$   
 vertex:  $(3, 15)$   
 axis of symmetry:  $x = 3$   
 domain:  $\mathbb{R}$  range:  $y \leq 15$



- 4) Write the equation of the line that passes through the points (6, 17) and (-3, 5). Then check your equation by plugging the points in and making sure they're on the line.

$m = \frac{17-5}{6-(-3)} = \frac{12}{9} = \frac{4}{3}$   
 $17 = \frac{4}{3}(6) + b$   
 $17 = 8 + b$   
 $b = 9$   
 $y = \frac{4}{3}x + 9$

- 5) Write an exponential equation that goes through the points (4, 31.1) and (10, 92.88). Then check your equation by plugging in the points by hand or in the calculator.

$92.88 = a \cdot b^{10}$   
 $31.1 = a \cdot b^4$   
 $\frac{92.88}{31.1} = \frac{a \cdot b^{10}}{a \cdot b^4}$   
 $2.99 = b^6$   
 $b = 1.20$   
 $31.1 = a \cdot 1.20^4$   
 $a \approx 15.0$   
 $y = 15(1.2)^x$

- 6) Write an exponential equation to model the population 'x' years after year zero. Then use your equation to complete the table. Write a scenario to fit the table.

x: # years after year zero	0	1	2	3	4	5
y: population	34499	29669	25,516	21943	18871	16,230

$16230 = a \cdot b^5$   
 $25516 = a \cdot b^2$   
 $\frac{16230}{25516} = \frac{a \cdot b^5}{a \cdot b^2}$   
 $\frac{16230}{25516} = b^3$   
 $b \approx 0.86$   
 $y = a \cdot 0.86^x$   
 $25516 = a \cdot 0.86^2$   
 $a = 34499$   
 $y = 34499(0.86)^x$



7) Given  $g(x) = x^2 - 7x + 10$  (do a) - d) algebraically, then check in  $y=$  and table)

a) Find  $g(0) = 0 - 0 + 10 = 10$  ✓  
 b) Solve for  $x$  if  $g(x) = 0$   
 $0 = x^2 - 7x + 10$   
 $\Delta = (x - 2)(x - 5)$   
 $x = 2 \quad x = 5$  ✓

c) Find  $g(18) = 18^2 - 7(18) + 10 = 208$  ✓  
 d) Solve for  $x$  if  $g(x) = 18$   
 $18 = x^2 - 7x + 10$   
 $0 = x^2 - 7x - 8$   
 $0 = (x - 8)(x + 1)$   
 $x = 8 \quad x = -1$  ✓

8) Solve. Check w/calculator.

a)  $2x^2 - 6x + 8 = y$   
 $x = \frac{6 \pm \sqrt{36 - 64}}{4}$   
 $x = \frac{6 \pm \sqrt{-28}}{4}$   
 no R solutions ✓

b)  $5x^2 + 3x = +10$   
 $5x^2 + 3x - 10 = 0$   
 $x = \frac{-3 \pm \sqrt{9 + 200}}{10}$   
 $x = \frac{-3 \pm \sqrt{209}}{10}$   
 $x \approx 1.15, -1.75$  ✓

c)  $x^2 + 2x - 1 = y$  (d)  
 $y = 7x + 5$   
 $y = 10x^2 - 21x + 9$   
 $x^2 + 2x - 1 = 7x + 5$   
 $x^2 - 5x - 6 = 0$   
 $(x - 6)(x + 1) = 0$   
 $x = 6 \quad x = -1$  ✓  
 $y = 47 \quad y = -2$  ✓  
 $(6, 47) \quad (-1, -2)$  ✓

9) Solve algebraically.

$-7x - 8y = -8.5$   
 $y = 6x - 23$   
 $-7x - 8(6x - 23) = -8.5$   
 $-7x - 48x + 184 = -8.5$   
 $-55x = -192.5$   
 $x = 3.5 \quad y = -23(3.5, 2)$  ✓

10) Solve each for 'm'.

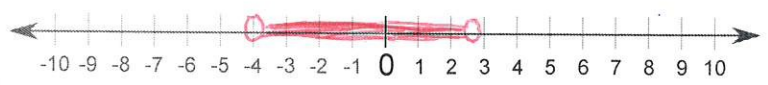
a)  $3m + 2b = 8$   
 $m = \frac{8 - 2b}{3}$

b)  $\frac{6g}{7m} = \frac{2m}{8v}$   
 $48gv = 14m^2$   
 $\pm \sqrt{\frac{24}{7}gv} = m$

c)  $8(2m - 3) = \frac{1}{2}$   
 $2m - 3 = \frac{1}{16}$   
 $2m = 3 \frac{1}{16}$   
 $m = 49/32$

11) Solve and graph your solution.

$-6 < -3m + 2 < 14$   
 $-6 < -3m + 2 \quad -3m + 2 < 14$   
 $-8 < -3m \quad -12 < 3m$   
 $8/3 < m \quad -4 < m < 4$   
 $m < 8/3$   
 $-4 < m < 8/3$



12) Write an equation that models the following scenario and use it to answer the question.

In 1950, a colony of mosquitoes is hit with a devastating disease and their population starts to decline. In 1952 its population is 325,954,125. In 1956 its population is 259,946,345. Write an exponential equation to model the population of the mosquitoes 'x' years after 1950. How many mosquitoes will there be in 1960? In what year will there be fewer than 80,000,000 mosquitoes?

$259946345 = a \cdot b^x$   
 $325954125 = a \cdot b^2$   
 $b \approx 0.945$

$325954125 = a(0.945)^2$   
 $a \approx 365,000,000$   
 $y = 365,000,000(0.945)^x$

$x = 10 \quad y = 365m(0.945)^{10}$   
 $\approx 207,305,560$  mosq.  
 $80000000 = 365m(0.945)^x$   
 $16/3 = 0.945^x$   
 $\log_{0.945}(\frac{16}{3}) = x$   
 $x \approx 26.8$  1976 ✓

13) Solve.

a)  $3 \cdot \sqrt{4x+3} = 33$   
 $\sqrt{4x+3} = 11$   
 $4x+3 = 121$   
 $4x = 118$   
 $x = 59 \frac{1}{2}$  ✓

b)  $7 \cdot x^{1/6} = 56$   
 $x^{1/6} = 8$   
 $x = 262144$  ✓

c)  $2 \cdot (x+3)^2 = 50$   
 $(x+3)^2 = 25$   
 $x+3 = \pm 5$   
 $x = 2, -8$  ✓

d)  $|2x-4| + 1 = 21$   
 $|2x-4| = 20$   
 $2x-4 = 20 \quad 2x-4 = -20$   
 $x = 12 \quad x = -8$  ✓

14) Solve.

a)  $9^x = \left(\frac{1}{27}\right)^{2x+1}$   
 $(3^2)^x = (3^{-3})^{2x+1}$   
 $2x = -6x - 3$   
 $x = -3/8$  ✓

b)  $625^{x-1} = 125^x$   
 $(5^4)^{x-1} = (5^3)^x$   
 $4x - 4 = 3x$   
 $x = 4$  ✓

c)  $\left(\frac{1}{8}\right)^{3x} = 128^x$   
 $(2^{-3})^{3x} = (2^7)^x$   
 $-9x = 7x$   
 $0 = 16x$   $x=0$  ✓

15) Simplify.

a)  $\sqrt{300x^2}$   
 $10x\sqrt{3}$

b)  $\sqrt{45}$   
 $\sqrt{9 \cdot 5}$   
 $3\sqrt{5}$

c)  $\sqrt{28} + \sqrt{63}$   
 $\sqrt{4 \cdot 7} + \sqrt{9 \cdot 7}$   
 $2\sqrt{7} + 3\sqrt{7}$   
 $5\sqrt{7}$  ✓

16) Rewrite the equations into Standard Form.

a)  $y = 5(x-3)^2 - 7$   
 $y = 5(x^2 - 6x + 9) - 7$   
 $y = 5x^2 - 30x + 38$  ✓

b)  $y = -2(x-5)(x-8)$   
 $y = -2x^2 + 26x - 80$  ✓

17) Rewrite the equations into Vertex Form by Completing the Square.

a)  $y = x^2 + 8x + 1$   
 $y = x^2 + 8x + 16 + 1 - 16$   
 $y = (x+4)^2 - 15$  ✓

b)  $y = 5x^2 - 20x - 11$   
 $y = 5(x^2 - 4x + 4) - 11 - 20$   
 $y = 5(x-2)^2 - 31$  ✓

18) Write an equation of a circle that has a center of  $(-6, 1)$  and a radius of  $\sqrt{5}$ .

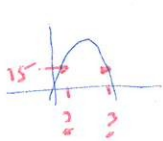
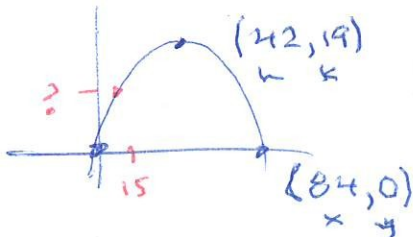
$$(x+6)^2 + (y-1)^2 = 5$$

19) Amazing Kitty jumps into the air in a parabolic path.

The horizontal distance covered by Kitty is 84 feet. The maximum height of Kitty is 19'. Sketch a picture of the situation and write an equation that models it.

Find Kitty's height 15' horizontally from start. Find horizontal distance for height: 15'.

Check in the calculator. Adjust windows so you can see the parabola & the given points.



$$y = a(x-h)^2 + k$$

$$0 = a(84-42)^2 + 19$$

$$-19 = a \cdot 1764$$

$$a = \frac{-19}{1764}$$

$$y = \frac{-19}{1764}(x-42)^2 + 19$$

$x=15$  height: 11.15'

$y=15$  horiz: 22.73' = 61.27'

20) Consider the graph of  $f(x) = \frac{1}{2}x^2 - 1$ . Shift  $f(x)$  to find new functions.

a) Write an equation to show that  $g(x)$  is shifted to the right and has no  $x$ -intercepts.

$$g(x) = \frac{1}{2}(x-3)^2 + 5$$

b) Write an equation to show that  $h(x)$  is steeper than  $f(x)$ .

$$h(x) = 4x^2 - 1$$

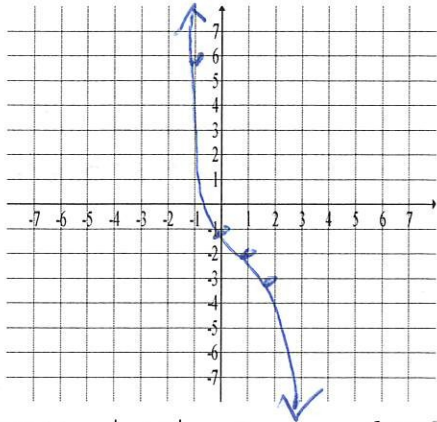
c) Write an equation to show that  $j(x)$  is facing down and is shifted to the left.

$$j(x) = -\frac{1}{2}(x+1)^2 - 1$$

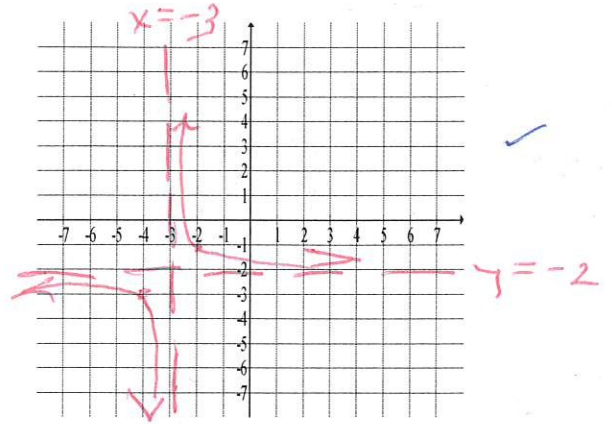


21-24) Name the families, compare the graphs to the parent graphs, and graph the functions. Find the domain and range. Also state the asymptotes if they exist.

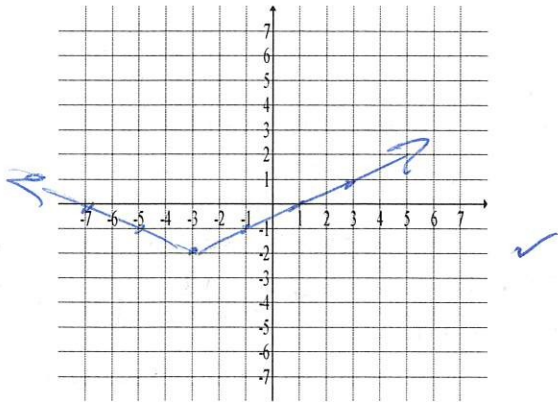
21)  $y = -(x-1)^3 - 2$  family: *cubic*  
 domain:  $\mathbb{R}$  range:  $\mathbb{R}$   
 a,h,k and impacts: *down Facing R1 D2*



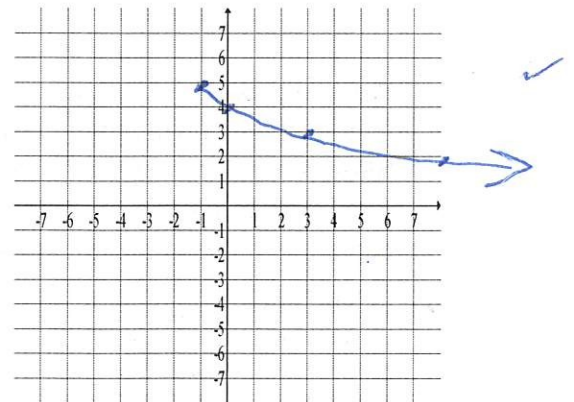
22)  $y = \frac{1}{x+3} - 2$  family: *Hyperbola*  
 domain:  $x \neq -3$  range:  $y \neq -2$   
 a,h,k and impacts: *2 3 D 2*



23)  $y = \frac{1}{2} \cdot |x+3| - 2$  family: *AbsValue*  
 domain:  $\mathbb{R}$  range:  $y \geq -2$   
 a,h,k and impacts: *wide 2 3 D 2*



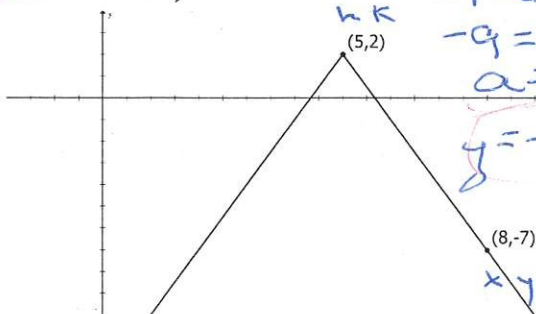
24)  $y = -\sqrt{x+1} + 5$  family: *Sq RT*  
 domain:  $x \geq -1$  range:  $y \leq 5$   
 a,h,k and impacts:



25) Write an **exact** equation for each graph. (you must show all of your work, as always!) Then check your equations by putting them in  $y=$  and checking for the given points.

25a

a)



$$y = a|x-h| + k$$

$$-7 = a|8-5| + 2$$

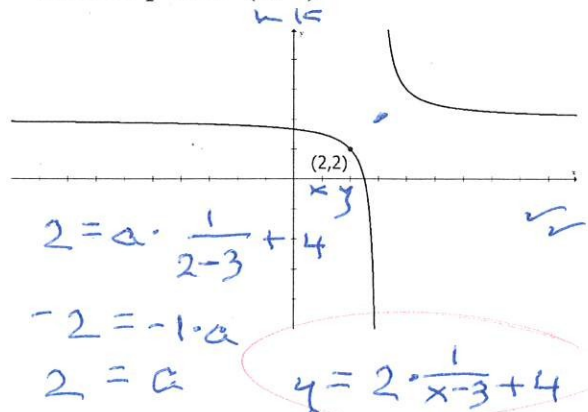
$$-9 = 3a$$

$$a = -3$$

$$y = -3|x-5| + 2$$

b)

locator point: (3, 4)



$$2 = a \cdot \frac{1}{2-3} + 4$$

$$-2 = -1 \cdot a$$

$$2 = a$$

$$y = 2 \cdot \frac{1}{x-3} + 4$$

- 26) Solve graphically.  
Do by hand and then check w/calculator.

(hint: use 4 colors)

$$y < (x+1)^2 - 8$$

$$-12 > -x - 2y$$

$$y > -3$$

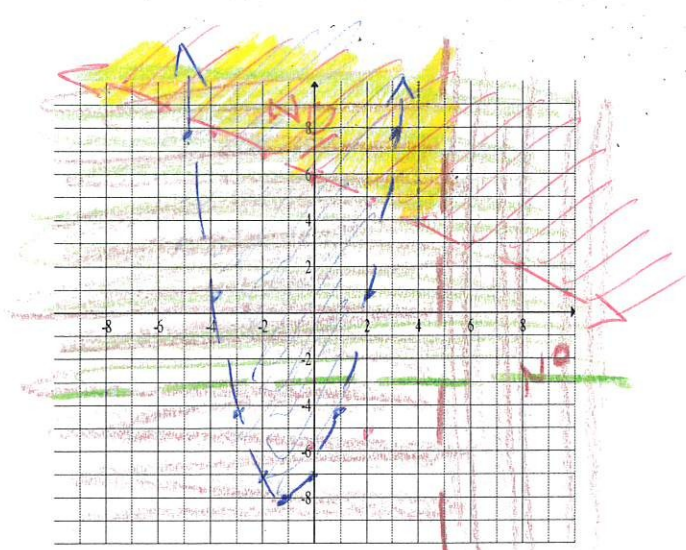
$$x \geq 5$$

$$-12 = -x - 2y$$

x	y
0	6
12	0

$$-2y = +x + 12$$

$$y = \frac{1}{2}x + 6$$



- 27) Solve by writing a system of equations and solving algebraically.  
Seven smoothies and twelve cookies cost a total of \$54.65.  
Twelve smoothies cost \$13.35 less than 27 cookies.  
Find the cost of a smoothie and the cost of a cookie. (check your solution!)

x: COST SMOOTHIE     $12(7x + 12y) = 54.65$   
y: COST COOKIE     $-7(12x = 27y - 13.35)$

$$84x + 144y = 655.8$$

$$-84x + 189y = 93.45$$


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$$333y = 749.25$$

$$y = 2.25$$

$$12x = 27(2.25) - 13.35$$

$$x = 3.95$$

SMOOTHIE \$3.95    COOKIE \$2.25 ✓✓

- 28) Solve algebraically. Explain what the solution tells you about the graph.

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$$y = x^2 - 7x + 14$$

$$y = x - 1$$

$$x^2 - 7x + 14 = x - 1$$

$$x^2 - 8x + 15 = 0$$

$$(x-5)(x-3) = 0$$

$$x = 5 \quad x = 3$$

$$x = 5 \quad x = 3$$

$$y = 4 \quad y = 2$$

$$(5, 4) \quad (3, 2)$$

intersection pts of line = parab ✓✓

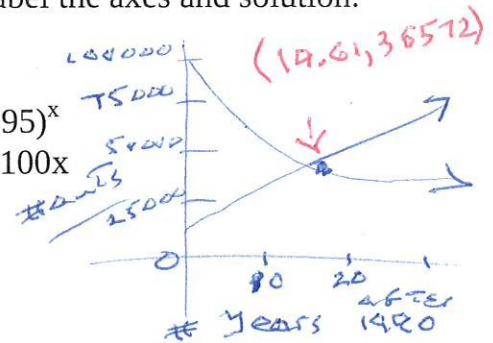
- 29) Given the following system of equations, describe the population of the ants over time. Then find when their populations will be the same (in the future) using your calculator. To show your work, draw a sketch of your solution. Be sure to label the axes and solution.

x: number of years after 1990  
y: population of ants in a hill

y<sub>1</sub>: Ant hill #1 - ↓ 5% / yr, 100,000 in 1990  
y<sub>2</sub>: Ant hill #2 - ↑ 1100 / yr, 15,000 in 1990  
~ 36572 in 14.61 years or 2009

$$y_1 = 100,000(0.95)^x$$

$$y_2 = 15,000 + 1100x$$



- 30) Find the inverses and then check by finding f(f<sup>-1</sup>(x)) or f<sup>-1</sup>(f(x)).

a)  $f(x) = \frac{3x^4 - 1}{5}$

$$x = \frac{3y^4 - 1}{5}$$

$$5x = 3y^4 - 1$$

$$5x + 1 = 3y^4$$

$$\frac{5x + 1}{3} = y^4$$

$$f^{-1}(x) = \pm \sqrt[4]{\frac{5x + 1}{3}}$$

$$f(f^{-1}(x)) = \frac{3\left(\frac{5x + 1}{3}\right)^4 - 1}{5} = \frac{5x + 1 - 1}{5} = \frac{5x}{5} = x$$

b)  $f(x) = \sqrt{3x - 7} + 2$

$$x = \sqrt{3y - 7} + 2$$

$$x - 2 = \sqrt{3y - 7}$$

$$(x - 2)^2 = 3y - 7$$

$$(x - 2)^2 + 7 = 3y$$

$$f^{-1}(x) = \frac{(x - 2)^2 + 7}{3}$$

$$f(f^{-1}(x)) = \sqrt{3\left(\frac{(x - 2)^2 + 7}{3}\right) - 7} + 2 = \sqrt{(x - 2)^2 + 7 - 7} + 2 = \sqrt{(x - 2)^2} + 2 = |x - 2| + 2 = x$$



31) Solve. and check!

a)  $\sqrt[5]{x^5} = 18 \quad x \approx 1.78$

b)  $3^{2x-3} = 30$

$\log_3 30 = 2x - 3$   
 $\frac{\log_3 30 + 3}{2} = x$   
 $x \approx 3.05$  ✓

c)  $\log_5 8 = 3x + 4$   
 $\frac{\log_5 8 - 4}{3} = x$   
 $x \approx -0.90$  ✓

d)  $2(5)^{x-5} - 3 = 55$   
 $5^{x-5} = 29$   
 $\log_5 29 = x - 5$   
 $\log_5 29 + 5 = x$   
 $x \approx 7.09$  ✓

e)  $\log(3) = \log(4) - 2\log(x)$

f)  $\log(4) + \log(50) = x + 1$   
 $\log(200) = x + 1$   
 $x = \log(200) - 1$   
 $x \approx 1.30$  ✓

$\log 3 = \log(4/x^2)$   
 $3 = \frac{4}{x^2} \quad 3x^2 = 4 \quad x^2 = \frac{4}{3} \quad x \approx 1.15$  ✓

32) a) Expand.

$\log\left(\frac{a^3 b}{c^{(1/2)} d^4}\right)$

b) Compress.

$5\log_8(m) - 3\log_8(n^2) - \frac{1}{2}\log_8(q^8)$

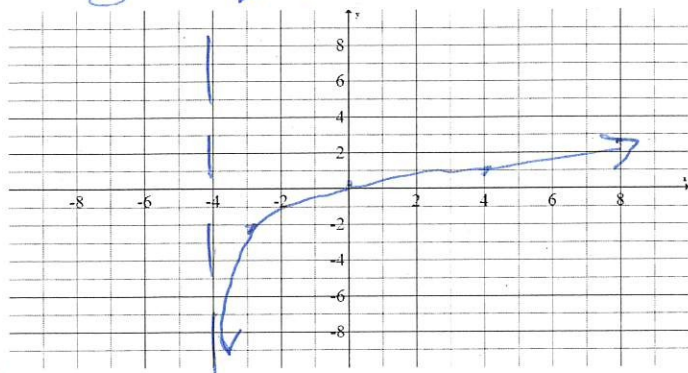
$3\log a + \log b - \frac{1}{2}\log c - 4\log d$

$\log_8\left(\frac{m^5}{n^6 q^4}\right)$

33) Graph  $y = \log_2(x+4) - 2$

Name 3 points on the graph and the equation of the asymptote.

asy:  $x = -4$   
 (0, 0)  
 (4, 1)  
 (-3, -2)



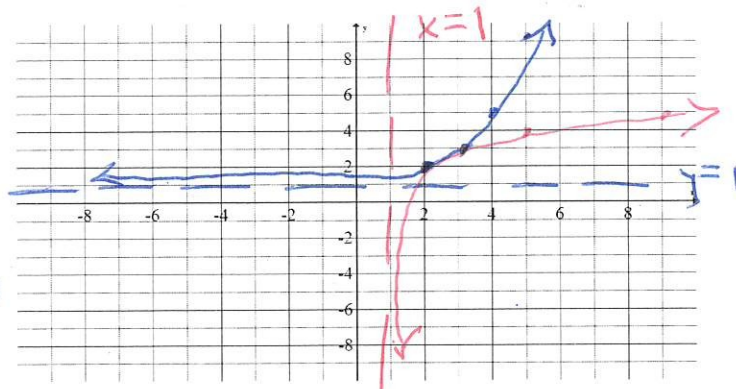
34) Find the inverse of  $f(x) = \log_2(x-1) + 2$ .

Graph  $f(x)$  and  $f^{-1}(x)$  and the line of symmetry.

Check your inverse by finding some points on  $f(x)$  and  $f^{-1}(x)$ .

$x = \log_2(y-1) + 2$   
 $x - 2 = \log_2(y-1)$   
 $2^{x-2} = y - 1$

$f^{-1}(x) = 2^{x-2} + 1$



35) Josie invests \$2000 in the stock market. It is gaining an average of 3.2% per year. Write an equation to model her investment. How much will she have in ten years? When will she have more than \$5000?

$y = 2000(1.032)^x$   
 $y = 2000(1.032)^{10}$   
 $\approx \$2740.48$

$5000 = 2000(1.032)^x$   
 $5/2 = 1.032^x$   
 $\log_{1.032}(5/2) = x$   
 $\approx 29.09$  years ✓

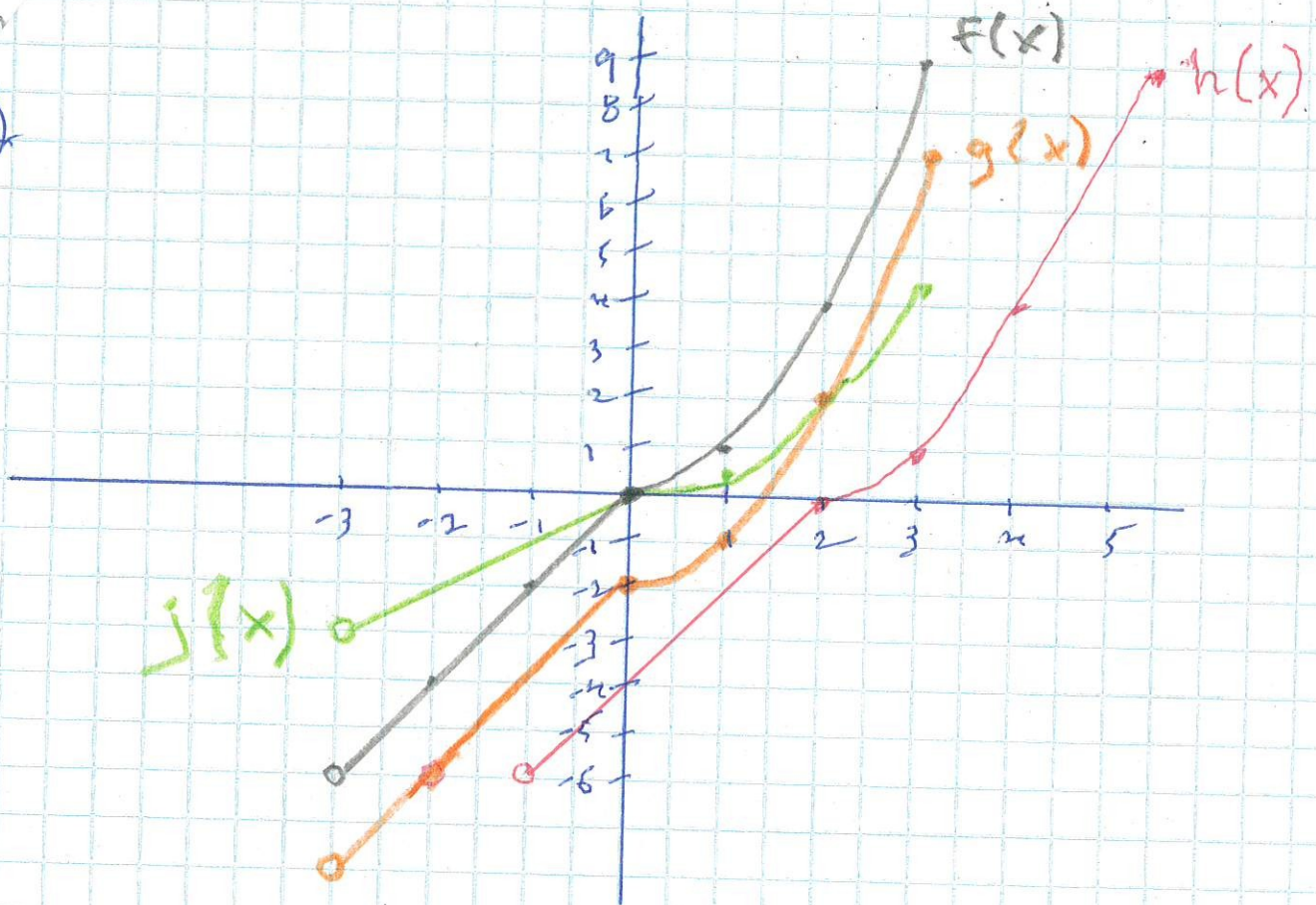
36) Joe invests \$2000 in a bank that pays him 3.2% compounded monthly. Write an equation to model his investment. How much will he have in ten years? When will he have more than \$5000?

$y = 2000\left(1 + \frac{0.032}{12}\right)^{12t}$   
 $y = 2000\left(1 + \frac{0.032}{12}\right)^{120}$   
 $\approx \$2753.08$

$5000 = 2000\left(1 + \frac{0.032}{12}\right)^{12t}$   
 $5/2 = \left(1 + \frac{0.032}{12}\right)^{12t}$   
 $\log_{1 + \frac{0.032}{12}}(5/2) = 12t$   
 $t = \frac{\log_{1 + \frac{0.032}{12}}(5/2)}{12}$   
 $\approx 28.87$  years ✓



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$$f(x) = \begin{cases} 2x, & -3 \leq x < 0 \\ x^2, & 0 \leq x \leq 3 \end{cases}$$

$$g(x) = f(x) - 2$$

DOWN 2

$$h(x) = f(x - 2)$$

RIGHT 2

$$j(x) = \frac{1}{2} f(x)$$

VERT. STRETCH:  $\frac{1}{2}$